Interferometry
For Accurate Displacement Metrology

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The most accurate measurements of displacements are made with optical interferometry. The author discusses some recent work in the field: a quadruple heterodyne Michelson interferometer for active position control and the use of Fabry-Pérot interferometry for displacements of up to 50 mm.

(Facing page) The interferometers described in the text are prototyped on a suspended optical breadboard in a vacuum chamber; the close-up (above) shows the acousto-optic modulators and fiber couplers used in the quadruple heterodyne interferometer.
The meter and its realization

Long gone are the days when the meter was defined by an artifact or as a fraction of the distance between the North Pole and the equator. The meter has been defined since 1983 as the distance light travels in vacuum in 1/299,792,458 seconds, from which follows the fundamental constant $c$, the speed of light. For laboratory-scale distances, however, the meter is usually realized not through a time-of-flight measurement but rather through the equivalent relation $\lambda = c / \nu$ for plane electromagnetic radiation in vacuum. In practice, this requires two things: a laser of well-known frequency $\nu$, and some sort of interferometer.

The available optical frequency standards are very good indeed. Iodine-stabilized helium-neon lasers, for example, with fractional accuracy at the level of $2.5 \times 10^{-11}$, have been commercially available for years. This level of accuracy corresponds to roughly one millimeter relative to the circumference of Earth, and standards with several orders of magnitude better accuracy are found at national measurement laboratories. In general it is the interferometer, not the laser, which limits the accuracy of a distance measurement.

While optics textbooks abound with interferometer designs of all types, we will focus here on only two: Michelson and Fabry-Pérot interferometers. In practice, the overwhelming favorite for the measurement of displacements is the Michelson interferometer. This type of interferometer is commercially available from a number of manufacturers, and widely used in laboratories and industry. The Fabry-Pérot interferometer has received far less attention in the field. Nevertheless, it offers some intriguing possibilities, including phenomenal resolution and a direct link to a time standard without the necessity of an optical frequency reference. Variations on these interferometers are in use in specialized applications.

The field of displacement metrology is broad, and comprehensive surveys exist elsewhere. Rather than attempt a complete overview, I will focus on recent work at the National Institute of Standards and Technology (NIST) aimed at increasing the accuracy of Michelson interferometry and exploring Fabry-Pérot interferometry.

**Two-beam (Michelson) interferometry**

The idea of two-beam interferometry is familiar, and is illustrated in Fig. 1(a). Light from a laser of known wavelength, $\lambda$, is split at a beam splitter and propagates to a fixed “reference” mirror and a movable “measurement” mirror. Upon reflection, it returns to the beam splitter and the reflections are superposed; a square-law detector then measures the output intensity. The intensity exhibits a sinusoidal dependence on the difference in length between the “measurement” and “reference” arms, as shown in the top of Fig. 1(c), with a period of half the optical wavelength, $\lambda/2$.

A practical instrument incorporates a number of refinements to this basic scheme. The mirrors are often replaced with corner cubes, making the measurement insensitive to deviations from rectilinear motion (tilts) as the target moves and preventing optical feedback to the laser. Sometimes, in order to increase the resolution, the light is made to take multiple reflections before being recombined; in this case, the period of the detected sinusoid may be $\lambda/4$, or smaller. In addition, the measurement and reference beams may be offset in frequency (making a “heterodyne” interferometer) in order to put information in a quiet region of the laser spectrum. Whatever the details, the principle remains: distance is inferred by counting the number of sinusoidal cycles measured at the detector, and if resolution better than $\lambda/2$ is desired, some means of fringe interpolation must be exploited. Alternatively stated, displacement is inferred by measuring a phase.

**Fundamental limit: photon statistics**

The only fundamental limit to measurement resolution is provided by shot noise in the photodetection process. This limit is, in fact, astonishingly small. Considering for concreteness an interferometer using only 1 mW of red laser light, and taking a measurement...
bandwidth of 10 kHz, the shot-noise limited resolution is $3 \times 10^{-13}$ m [Ref. 3], corresponding to $10^{-9}$ optical fringes. Remarkably, it is entirely realistic to construct a system in which the dominant source of statistical error is shot noise.

The key to obtaining such performance is that laser amplitude noise is shot noise limited at high frequencies. With a typical helium-neon laser operating at 633 nm, the shot-noise limit is attained at frequencies beyond 700 kHz [Ref. 4]. Shot-noise limited signals, then, can be achieved by encoding information in this range. With a Michelson interferometer, this encoding may be accomplished by choosing a heterodyne frequency above 700 kHz.

Fringe interpolation errors

In practice, the main source of measurement error is not statistical, but rather systematic, and occurs in the process of interpolating the fringe. (An additional systematic error arises from the fact that the relation $\lambda = c/\nu$ is only an approximate one for laser beams, which because of their finite cross-section are not plane waves. One can, however, use the exact relation for Gaussian beams; further discussion of this topic is beyond the scope of this article and interested readers are encouraged to consult Ref. 5.) An obvious question arises: how does one measure fringe interpolation errors without having some “reference” measurement system known to be better than the interferometer? One clever approach has been in use for a long time. The idea is simple: if, for whatever reason, the interferometer makes a systematic error in fringe interpolation, the error will be repeated on every fringe. Thus, by moving the measurement mirror over a large number of fringes, one will obtain data with periodic residuals indicative of the interpolation error.

We at NIST and others have experimented with heterodyne interferometers in which the measurement and reference beam frequencies are generated by means of two acousto-optic modulators (AOMs). With such systems we have obtained periodic systematic errors at the level of about 1/36,000 fringes. To achieve this level of performance, it is important to pay careful attention to optical feedback and stray reflections.

Real-time position control

In Fig. 2, we show how these ideas can be developed further to create a quadruple interferometer designed to simultaneously measure displacement, pitch and yaw. Only two of the four interferometers are shown; the other two are in a plane parallel to the page. The beams comprising one interferometer are shown in bold. (b) Quasi-monolithic beam splitter to generate eight parallel beams from two incoming beams. The spacers are made of zerodur, and three different coatings are employed.

![Figure 2. (a) Layout of a quadruple heterodyne Michelson interferometer designed to measure displacement, pitch and yaw. Only two of the four interferometers are shown; the other two are in a plane parallel to the page. The beams comprising one interferometer are shown in bold. (b) Quasi-monolithic beam splitter to generate eight parallel beams from two incoming beams. The spacers are made of zerodur, and three different coatings are employed.](image)
used in the measurement. Since the four signals are at different frequencies, they can be electronically summed inside the vacuum chamber and brought out on just two coaxial cables (one reference and one measurement) for subsequent demodulation. The whole system is designed to be used inside a real-time servomechanism and to allow active control of displacement while nulling excursions in pitch and yaw.

**Multiple beam (Fabry-Pérot) interferometry**

An interesting alternative to the two-beam Michelson interferometer is provided by multiple-beam interference in a Fabry-Pérot interferometer. Fig. 1(b) shows the simplest such interferometer possible for length measurement; the signal resulting from a displacement is shown in the bottom half of Fig. 1(c). As in the Michelson interferometer, peak intensity is obtained every time the cavity length changes by \( \lambda/2 \); in this case, however, the detected signal is not a sinusoidal function of the displacement. The distinguishing feature of the Fabry-Pérot interferometer is the narrow resonance peak, which may be thousands of times smaller than the optical wavelength itself. The ratio of the spacing between adjacent peaks to the width of an individual peak is the cavity finesse. The finesse is set exclusively by cavity losses and is approximately 2,000 in the work described here.

The Fabry-Pérot interferometer thus offers the significant advantage that nature does the fringe interpolation; in other words, interpolation by a factor of the finesse comes “for free,” and of course one can go further and interpolate the narrow resonance peak. As shown, however, the system of Fig. 1(b) is of limited use, since if the cavity length is not exactly at resonance with the light, there is no signal at all. [For certain applications, this is not a problem; such a system has been used at NIST to define steps of \( \lambda/2 \) and—in conjunction with an X-ray interferometer—to measure the lattice constant of silicon.7]

A more generally useful way of extracting information from a Fabry-Pérot cavity is to use a tunable laser, as shown in Fig. 3. The key point here is that the cavity acts as an optical bandpass filter, the transmission frequencies of which are intimately related to the cavity length. If a laser is tuned to be resonant with the cavity and the cavity length is changed, the laser frequency must be changed as well in order to maintain resonance, and the change in length \( dL \) can be inferred from the change in laser frequency \( d\nu \). The relation, \( dL = \frac{c}{2 \nu} N d\nu \), requires knowledge of the optical order number \( N \) as well. \( N \) is the number of half-wavelengths resonant within the cavity; I will show below how it can be measured. Another advantage of the Fabry-Pérot interferometer is therefore that the measurement problem is reduced to that of measuring a frequency, rather than a phase—and frequency measurement is something that physicists are rather good at.

**Bay’s method: direct link to a time standard**

A problem arises, however, when the length change \( dL \) induces a frequency change \( d\nu \) sufficiently large that the frequency of the mode being probed passes out of the tuning range of the laser. A solution that employs the periodic frequency response of a Fabry-Pérot cavity is embodied in a suggestion made in 1970 by Zoltan Bay (at the time, Bay worked at the National Bureau of Standards, which later became NIST).8 Bay’s idea was to probe two cavity modes rather than one, and to measure the frequency difference between modes. The implementation he envisaged is shown in Fig. 3(b). Phase-modulation sidebands are introduced to laser light by means of an electro-optic modulator (EOM) and the sidebands are independently locked to two distinct cavity modes. (The necessary two degrees of freedom are obtained by adjusting the EOM drive frequency and the laser frequency). The EOM frequency is denoted by \( \Delta_1 \) and the integral number of cavity modes spanned by the two resonant sidebands by \( \Delta N_f \) (some additional information is required to obtain \( \Delta N_f \), such as an approximate idea of the cavity length). The length of the cavity is
changed, the measurement is repeated, and the displacement can be shown to be given by

\[ \Delta L = \frac{c}{4} \left[ \frac{\Delta N_f}{\Delta f} - \frac{\Delta N_p}{\Delta f} \right]. \]

Remarkably, the change in length is given exclusively by the two rf measurements, \( \Delta f \) and \( \Delta f \), the two integers \( \Delta N_f \) and \( \Delta N_p \), and the fundamental constant \( c \). It is especially noteworthy that the laser frequency does not enter into this equation. This approach has a certain aesthetic appeal since it is tied only to the time standard used for the rf measurements, thus obviating the need for an optical frequency standard. It is also the case that the measurement does not need to be corrected for the phase shift arising from the finite cross-section of the laser beam mentioned earlier.

Our own implementation of Bay’s idea is somewhat different; it is shown in Fig. 4 (left). Light from a helium-neon laser at 633 nm is split into two beams, one of which is blue-shifted and one of which is red-shifted by independently tunable AOMs. In this way, it is possible to lock the AOM outputs to adjacent cavity modes \( (\Delta N = 1) \) and measure the optical frequency difference very precisely by summing the rf oscillator frequencies. In addition, the absolute frequency of the laser light is measured by continuous comparison with an iodine-stabilized laser. The setup in our laboratory is designed to measure a cavity the length of which can change from 180 mm to 230 mm.

**Cavity interrogation and order numbers**

In this case as well, we desire to put information above 700 kHz in order to work in a region where the laser is shot noise limited. To this end the well known “Pound-Drever-Hall” scheme of cavity interrogation is used. A typical resonance curve is shown in Fig. 5. Peak-to-peak displacement corresponds to a length change of about \( \lambda / 4,000 \), or 160 pm. When the system is locked, the error is driven to zero with an rms deviation of about \( 1/1,000 \text{th} \) of the linewidth in a 10 kHz bandwidth.

This level of performance allows us to actually measure the exact order number \( N \) without ambiguity. \( N \) is given by the ratio of the optical frequency \( v_N \) of the mode (obtained by beating the light interrogating the cavity against an iodine-stabilized laser) divided by the difference of the optical frequencies between adjacent modes (measured via the sum of the AOM frequencies in Fig. 4). A small correction must be made because of frequency-dependent phase shifts at the mirrors, but the details are not of interest here. In this way we can distinguish, for example, a cavity containing 654,321 half-wavelengths and one containing 654,320 half-wavelengths. As it happens, this requires finding the centers of the resonances in Fig. 5 to approximately one part in 6,000. Achieving this level of accuracy is very challenging. Not only is it important to have good control of vibration and technical noise such as that caused by ground loops, it is essential to suppress all parasitic Fabry-Pérot cavities caused by stray reflections.

**Limited by frequency standard**

Thus, we have two ways to measure displacements with this system. In one approach, we use Bay’s idea directly, and obtain

\[ \Delta L = \frac{c}{4} \left[ \frac{1}{\Delta f} - \frac{1}{\Delta f} \right]. \]

This measurement does not involve an optical reference at all, and is derived from the time base in the counter used to measure the frequency intervals \( \Delta f \) and \( \Delta f \). In a second approach, we obtain the order numbers \( N_f \) and \( N_p \) as discussed above, and combine this information with measurements of the shift in optical frequency, as shown in Fig. 3(a). In this way we obtain a measurement in which the dominant source of uncertainty is the iodine-stabilized laser itself.

**Many wavelengths without counting fringes**

Another advantage of the Fabry-Pérot approach is also apparent: a displacement measurement can be performed without the need to continuously interrogate the cavity. Global displacement information to within \( \lambda / 2 \) is contained in the measurements of the order numbers \( N_f \) and \( N_p \), and it is not necessary to count fringes as the target is moved. Indeed, fringe counting may be impossible for a host of reasons, for example if the target speed is even momentarily high enough that the fringe frequency exceeds the electronic measurement bandwidth.

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Bay’s method “on steroids”: the frequency comb

The real power of Bay’s method is not, however, realized in the scheme discussed above. To obtain high resolution, one seeks not just to look at adjacent modes but to span a great many Fabry-Pérot resonances. Recently, optical frequency metrology has been revolutionized by the use of frequency combs based on modelocked femtosecond lasers.10 Below we describe very briefly ongoing research designed to use this technology to address the displacement metrology problem.

As shown in Fig. 4, we supplement the two frequencies at 633 nm which are used to probe modes $v_N$ and $v_{N+1}$ with an additional (orange) helium-neon laser at 612 nm to probe mode $v_{N+M}$. When all three frequencies are resonant with the cavity, the periodic cavity resonance condition in conjunction with a priori information about the frequency of the orange laser allows $M$, the number of modes spanned, to be determined. Meanwhile, the frequency difference between mode $N$ and mode $N+M$ is independently measured by beating the light probing them against light from a visible frequency comb, as shown in Fig. 6. Preliminary data have been obtained and are encouraging.

Conclusion

The link between a laboratory frequency standard and a physical displacement is made with an interferometer. The optimal design for a particular application involves many issues, including the level of accuracy required, range, speed, environment, allowed cost and complexity. I have focused here on recent work at NIST, in particular the little-explored field of Fabry-Pérot displacement interferometry. In so doing, I have necessarily omitted discussion of many relevant topics. It is my hope, however, that this contribution will be a stimulating source of ideas.

References