Fancy footwork: Understanding funhouse mirrors

The Reuben H. Fleet Space Theater and Science Museum has a number of displays for interactive use by children. One of the most puzzling (to children and adults alike) is the so-called "footwork" mirror. This small display on the second floor consists solely of a mirror curved around horizontal axes into a shallow "S" shape. It is placed at the wall so that, upon passing, one sees a short stubby leg hanging isolated in space with a foot on the top facing upward and another on the bottom more or less normally situated.

The funhouse mirror is a very complex item and understanding it involves setting aside disbelief to the extent that horizontal things (images) can be imagined in one place and that vertical things are somewhere else; and, worse yet, that these horizontal and vertical things are part and parcel of the same identical thing—the image of the actual object.

To understand how the footwork mirror or any peculiarly shaped mirror works, one needs to keep in mind a few simple aids to understanding. An effective way to talk to youngsters about mirrors is to describe them as if they were windows into another, sometimes very odd, world. The edges of the mirror constitute a frame. What we see in that frame we see by virtue of light rays that obey very simple rules:

- Light rays always travel in straight lines.
- When they reflect, it's always at an angle equal to the angle with which they strike the mirror. For all intents and purposes, each ray can be considered to strike a very tiny plane mirror imbedded in the surface of the entire mirror.
- It is not possible to "see" an object, or at least a small part of an object, unless the light ray that reveals that detail to our senses begins at the detail and eventually reaches our eye. The path it follows must obey the above two rules.

Here is how we can apply these rules to viewing ourselves in a mirror, for instance: First imagine that from our eye we can draw a number of straight lines toward the mirror representing the paths of the incoming rays as reflected from the mirror. Since we know the above rules, we can extend those lines by "backward" reflection and see where they came from. This identifies the details of the image we see. It gives only the direction of the detail in question and not its location in space. It is important not to confuse knowing the direction of the image details with knowing their exact location in space. Remember that the mirror is a frame or window and the brain will always tell you that the image we see is behind the mirror even when, in actual fact, it may as easily be in front. Keep in mind that these straight lines as they strike the eye tell us the directional location of the image details. We cannot reconstruct an image from this information alone. We need more.

To find out where the image is in space, we need to consider the divergence of the beam. Here the curvature of the mirror comes into play. A small cone of rays diverging from some point on the subject's body—his foot, say—will strike the mirror, each of the many rays obeying the same rules. But since they may strike a curved surface of some finite size, now their divergence can be affected by the curvature of the mirror. The curvature of the mirror may increase the divergence, decrease it, or even reverse it to a convergent bundle—all this in addition to reflecting the bundle in accordance with our rules. We can determine where those rays appear to come from by extending their direction backwards: this is where we will place the image in space. All of this is very straightforward for axially symmetric mirrors: plane, convex, or concave.

In plane mirrors, we always see our image as standing behind the mirror just as far

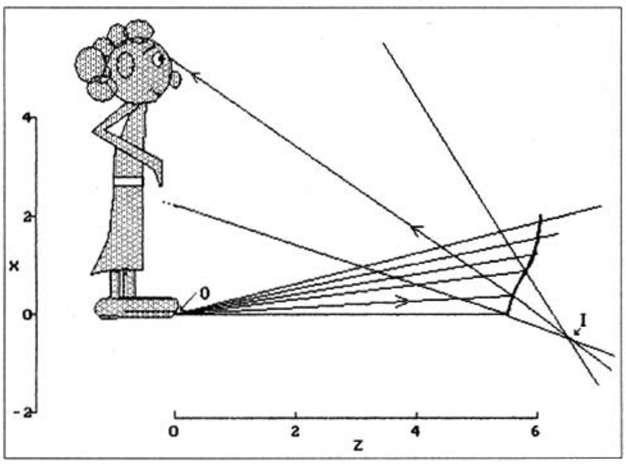

Figure 1. Convex cylindrical mirror ray trace

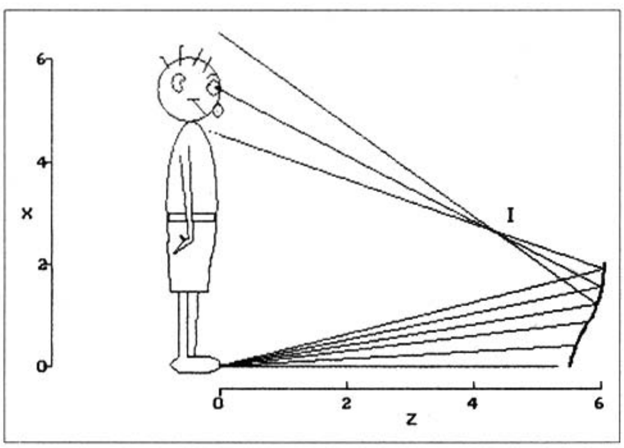

Figure 2. Concave cylindrical mirror ray trace

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back from the plane of the mirror as we are in front of the mirror. This image we describe as virtual, since no light really exists behind the mirror surface. It is, of course, also erect. Drawing the lines as described easily shows this symmetry.

For convex mirrors, the diverging cones of rays are caused to diverge even more by the mirror surface and we see a tiny image still behind the mirror surface but very close at hand. The garden globe is the common example of this mirror wherein the entire surrounding area is apparently imaged in miniature inside the globe.

Concave mirrors are a little more complex. We normally view ourselves in these fairly close at hand. Then they appear to magnify. The image appears to be behind the mirror and is erect. Again it is virtual, as no light is actually present at the perceived location of the image. If we move quite far back from the mirror, we will finally see our image upside down. We may think the image is behind the mirror since the mirror still forms a frame for it, but it is not. It is in front of the mirror. It is real in that light rays forming a detail actually converge at that point in the image. If we were to place a small slip of paper at the image, we would see the image projected on it as on a movie screen.

The fun house mirror or the footwork mirror presents us with a real puzzle. But the puzzle is not what most people think it is. Most people might say, "Why do I look fat?" or "thin" or "How come my leg has a foot on top as well as on the bottom?" The real puzzle is: "Where is the image?"

If a mirror is bent vertically, but not horizontally, image point locations will be in two different places depending on whether we look at a vertical fan of rays from an object point or a horizontal line fan. The footwork mirror is not bent horizontally; thus, details spaced horizontally will be imaged as in a plane mirror, that is, behind the mirror at an equal distance behind as the subject is in front. Details spaced in the vertical direction, however, will obey the law of curved mirrors and will either be compressed vertically (convex) or spread out and magnified (concave). If the location of the object is sufficiently far away and the curvature is concave, the vertical row of details will appear inverted.

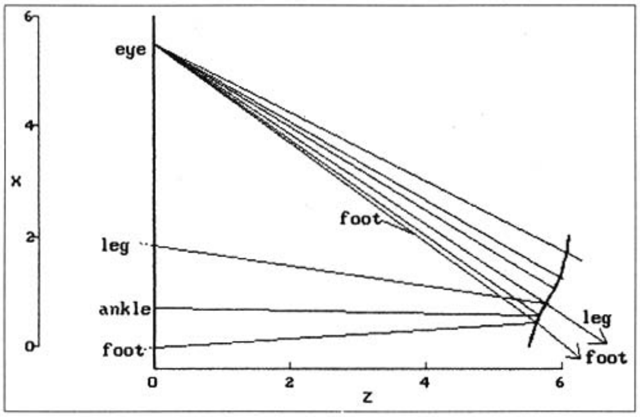

Figure 3. Ray trace to eye from foot and leg: Convex cylindrical mirror

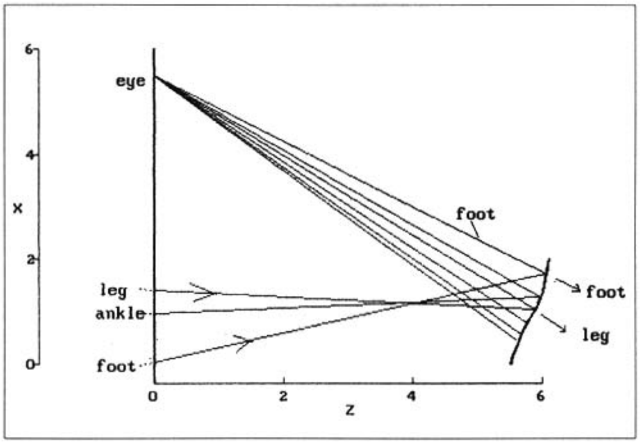

Figure 4. Ray trace to eye from foot and leg: Concave cylindrical mirror

In Figures 1-4, generated by an automatic lens design computer program, we see:

1. A cone of rays emanating from the "foot" intercepting the convex part of the mirror and showing the virtual image point behind the mirror. Note that it appears close to the mirror and below floor level, in fact. Check this by direct observation.

2. A cone of rays emanating from the "foot" intercepting the concave part of the mirror and forming a focus in front of the mirror as well above the floor. If one considers a point just below the foot, this would be imaged even higher, so we conclude this "foot" image is inverted.

3. A cone of direction lines emanating from the eye strikes the convex mirror at various points and by reflection show the position of the leg, ankle, and foot as seen on the "window" of the mirror. Note they are in erect order, foot below and leg above.

4. A cone of direction lines emanating from the eye strikes the concave mirror at various points and on reflection cross over and indicate the position of leg, ankle, and foot. Now we see the "foot" ray is the topmost ray.

The top half of the footwork mirror is concave. In this half, we can see a foot on top and a leg extending down from it. As our line of sight progresses down the mirror, it encounters the lower portion which is convex rather than concave and mirrors the leg and foot in an upright position (although compressed in the vertical direction) and quite logically connected to the image above.

The peculiar part and the real question is, "Where is the image?" The image does not have a location in any real sense. In the science of optics, we would say that the image is highly "astigmatic." This simply means that horizontal fans of rays come to a focus at a different distance than do vertical fans. The eye and brain are enormously forgiving. Lenses can form very bad images that the eye and brain say look okay. Our brain fills in an awful lot if the subject viewed is familiar. Remember that the edge of the mirror is a frame for the picture and so the brain says "the image is behind the mirror—all there together as it should be. My leg and foot are certainly not somehow disconnected and spread out in space!"