The realization of Bose-Einstein condensation in dilute gases in 1995 provided the atom analogue to laser light. Just as coherent high-intensity laser sources opened many new areas for research, Bose-Einstein condensation has opened up many analogous areas of coherent atomic physics. In particular, just as laser light enabled the development of nonlinear optics, in which a light field in a medium interacts with itself through mediating forces arising from the atoms in the medium, Bose-Einstein condensation has enabled the complementary field of nonlinear atom optics. A Bose-Einstein condensate (BEC) is a coherent atom field made up of very cold localized atoms. Although we usually think of atoms as interacting by collisions, these collisional interactions are mediated by electromagnetic fields. Thus nonlinear interactions in a BEC—the coupling of the atom field with itself via electromagnetically mediated atomic interactions—is the exact complement to nonlinear optics. For this reason it has been possible to demonstrate the counterpart to such classic nonlinear optics experiments as four-wave mixing with the matter waves of BECs.

One distinction between laser light and a BEC is that the nonlinear interactions in a BEC are inherently much larger, so the self-interactions greatly affect the properties of all but the very lowest-density BECs. In this article we will briefly review a few recent experiments investigating novel behaviors in BEC arising from or depending on nonlinearities, particularly those with analogues in optics. We first review basic concepts important to understanding BEC experiments. We then discuss the nature of the atomic interactions, and describe experiments in which attractive and adjustable interactions play a dominant role in the dynamics of the condensates. Finally, we review some recent experiments in which the BEC atoms repel each other, permitting the existence of stable vortices in condensates.

Overview of Bose-Einstein condensation

When an isolated sample of identical atoms in a gas is cooled to extremely low temperatures, a fraction of the atoms end up in the ground state of the confining potential. Confinement is usually achieved in a magnetic trap. This macroscopic occupation of the ground state is known as Bose-Einstein condensation, and the resulting condensate provides a fascinating system where quantum-mechanical behavior of the system as a whole is evident on macroscopic size scales. A BEC in a dilute gas can be thought of as a dilute superfluid, or the matter equivalent to laser light: the constituent atoms are monoen-ergetic and phase coherent. In fact, the entire group can be considered as one macroscopic quantum entity that can be mathematically represented by a simple quantum wave function. Depending on the trap geometry, BEC temperatures can be as low as a few nK, which is nine orders of magnitude colder than any naturally occurring matter in the universe!

Experimentalists are now able to cool atoms to the BEC transition temperature in a few dozen laboratories around the
world, but creating a condensate is still quite challenging. The $^{85}\text{Rb}$ BEC experiment at JILA is a case in point. Figure 1 is a picture of the vacuum chamber in which BEC is produced. The evacuated glass cell in which the $^{85}\text{Rb}$ atoms are magnetically trapped and cooled is barely visible, half hidden by 11 electromagnetic coils and several optical elements required for the cooling and imaging. The entire apparatus (not shown) takes up two optical tables and requires the light from five different lasers.

The role of interactions

Although the density of the atoms in an atomic BEC ($\sim10^{14}/\text{cm}^3$) is typically five orders of magnitude lower than the density of air, the interatomic interactions dramatically affect nearly all BEC properties. These include static properties like the BEC size and shape, and dynamic properties like the collective excitation spectrum and soliton and vortex behavior.

The role of interactions in a BEC can be appreciated by looking at the nonlinear Schrödinger equation (NLSE) that is typically used to model BEC behavior:

$$i\hbar \frac{\partial \Psi}{\partial t} = -\frac{\hbar^2}{2m} \nabla^2 \Psi + V + u|\Psi|^2 \Psi.$$  \hspace{1cm} (1)

This equation is analogous to the nonlinear equation that describes the behavior of an electromagnetic field in nonlinear optics through a term proportional to the intensity of the field. The condensate is represented by the macroscopic wave function $\Psi$. The first two terms in parentheses are the kinetic and potential energy terms. The nonlinearity is accounted for by the third term, which is proportional to the square of the amplitude of the wave function representing the atom density distribution. The proportionality factor is given by $u = 4\pi n^2 a/\hbar^2$, where $m$ is the atomic mass and $a$ is a parameter called the $s$-wave scattering length, which is determined by the atomic species and the internal atomic spin state.

When $a > 0$, the atoms repel each other and the steady-state BEC size is larger than for the noninteracting case. In contrast, when $a < 0$, the interactions are attractive and a BEC contracts to minimize its overall energy. The nonlinear term in the NLSE thus predicts BEC behavior similar to optical "self-defocusing" for condensates with repulsive interactions, and "self-focusing" when the interactions are attractive. In a trap, the contraction for attractive interactions competes with the "quantum pressure" from the quantum kinetic energy arising from the uncertainty principle, which tends to enlarge the BEC. For strong enough attractive interactions or for high enough atomic densities in an attractive system, the quantum pressure is insufficient to stabilize the BEC. The NLSE then has no steady-state solution, and the BEC collapses in on itself. A BEC can avoid such an implosion only as long as its self-interaction energy is not too large, or equivalently, as long as the number of atoms it contains is less than a critical value that is inversely proportional to $a$.

Figure 1 The false-color image in the center is a reproduction of a BEC. The false-color image on the left is a close-up view of the BEC. The false-color image on the right is a wide view of the BEC. (The top of the cube is a reproduction of a BEC.)

Figure 2 A "Bosenova" (see p. 34). The false-color images are each 130 x 80 µm, and the sequence covers a time scale of approximately 5 ms after turning off the magnetic trap. The burst of atoms emerges as an expanding halo (shown in dark blue), leaving a cold remnant condensate behind in the center.

Figure 3 A filled BEC vortex. The density distribution of a BEC is measured by using a probe laser beam to image the BEC. Because a BEC is confined in three dimensions—in contrast to optics experiments—imaging along different directions provides information about the topological structure of the BEC. Here, the side and front of the "cube" display two views of a filled vortex in a $\sim50$-µm-wide BEC. (The top of the cube is a reproduction of the side view.) The front view looks down the vortex core and determines its position, while the side view looks at the side of the core.
repulsive condensates with up to 15,000 atoms, and then to manipulate the interatomic interactions of these condensates by applying magnetic-field ramps.

Experiments using these techniques verified that the onset of instability was indeed inversely proportional to the scattering length as theoretically predicted. When the scattering length is stepped to a value more negative than the critical value determined by the stability criterion, theory predicts that a condensate’s kinetic energy is no longer sufficient to stabilize the condensate, and it collapses on itself. Indeed, when the interactions were suddenly changed from repulsive to attractive in the experiment, a violent collapse of the condensate was observed. The level of control of the onset of instability allowed for detailed studies on the dynamics of the collapse. On a time scale of a few milliseconds after switching the interactions from repulsive to attractive, an explosion occurred in which ~20% of the atoms were ejected from the condensate in a detectable “burst” of atoms that then remained in the magnetic trap. The atoms in the burst had energies that were much larger than the energy per atom of the initial sample (~3 nK), but much smaller than the depth of the magnetic trap potential. A sequence of images of such a burst is shown in Fig. 2.

There is a surprisingly close analogy between an exploding condensate and a supernova—so close, in fact, that the BEC explosion was dubbed a “Bosonova.” Although a supernova is typically 15 orders of magnitude larger in size and 73 orders of magnitude larger in energy than its quantum counterpart, both Bosonovae and supernovae occur when the balance of forces governing their size and shape is suddenly altered. Both exspel matter in dramatic explosions, and both leave behind remnants. The remnant condensates left behind after a Bosonova contain some fraction of the initial atoms and oscillate in a highly excited collective state. One mystery of the collapse process is that a substantial fraction (40 – 70% depending on conditions) of the sample atoms is missing after a collapse. What happens to these missing atoms is still unknown. They could be converted to undetected internal states or molecules, or they could be ejected from the condensate with energies so high that they are not detected.

Vortices in a quantum fluid

In contrast to attractive interactions, repulsive interactions stabilize a trapped BEC, enabling studies of the creation and dynamics of topological features such as quantum vortices. Quantized vortices in BEC provide further common ground between optics and atom optics. They also provide a link with research in superfluid helium, superconductivity, and even the rotation of neutron stars. In these quantum systems, a vortex is a topological singularity in a fluid created and stabilized by fluid flow around the singularity. The vortex flow pattern is irrotational, that is, it is curl free, and consequently the local fluid velocity is fastest near the singularity, also called the vortex core. Right at the singularity, the fluid velocity would be infinite. However, fluid is absent from this point and the flow pattern remains well defined throughout the fluid.

In a superfluid such as a BEC, the quantum phase must be single-valued at any point in the fluid. Thus the phase can only change by integer multiples of 2π around any closed loop encircling a phase singularity. This is precisely the condition that leads to quantization of angular momentum for a superfluid and an optical beam. Vortices in 87Rb BEC were first reported at JILA in 1999,13 and in the last two years, vortex experiments have been performed in a number of groups. Here we will briefly review these experiments.

Filled vortex cores

The first successful observation of vortices in BECs used a BEC phase and density engineering technique. From a condensate made of one internal hyperfine spin state, a condensate of a different spin state was created having the phase dependence of a vortex. Thus a singly quantized vortex was formed in just one of the two superimposed BEC spin components. The other (non-rotating) spin component filled the vortex core of the rotating component. This scenario is reminiscent of nonlinear optics experiments in which the core of an optical vortex is used as a waveguide for another beam of light.14 The core-filling component of the BEC vortex could be slowly or quickly removed with a properly tuned laser beam, allowing the experimenters to study a continuum of possible configurations between filled and empty vortex cores. Without the filling, the nonlinear interactions of the BEC determine the size of the core, restricting it to dimensions usually smaller than 1 μm, making vortex cores difficult to observe optically. However, in the presence of the core-filling component, the core was large enough to be observed in situ, as shown in Fig. 3. A second related consequence of interactions is that stable vortex cores are only permitted for repulsive interactions; in an attractive system, the BEC minimizes energy by expelling vortices and acquiring center-of-mass angular momentum.

Of course, it is possible to have density distributions with holes in them that are

Figure 4. Phase measurement of a BEC vortex. The vortex (a) and the core-filling component (b) interfere, producing an interferogram (c), shown for one of the two BEC spin states. (d) Colors show the measured values of the cosine of the phase difference between the vortex and core filling obtained through analysis of (a), (b), and (c), indicating the presence of a 2π phase winding in all images, θ labels an azimuthal reference angle, not the quantum phase. Adapted from Ref. 13.
not vortices. To confirm that the BEC had the phase character indicative of a vortex, an interferometric technique similar to optical phase measurements was used. A vortex, shown in Fig. 4 (a), was interfered with a constant-phase BEC component trapped in the vortex core, shown in Fig. 4 (b), which served as the constant-phase “reference” beam. A microwave pulse served as the “beamsplitter” (depicted as a gray bar in Fig. 4 (d)) that coupled the two condensates to convert phase into condensate amplitude. An atom-density measurement of one of the two spin components, Fig. 4 (c), revealed regions of constructive and destructive interference in the areas where the vortex and filling overlapped. This interference pattern was a direct signature of a 2π phase winding, a singly quantized vortex.

Another benefit to having the filled vortex core is that it makes the core large enough to be directly observed. This allowed the JILA team to observe the precession of an off-centered vortex core around the center of the BEC, an inherent part of vortex dynamics in a confined fluid. The vortex dynamics could also be manipulated by adjusting the magnetic trapping potential, an ability unique to BEC systems.

Vortex lattices

Not long after the creation of these single vortices, Jean Dalibard’s group at the École Normale Superieure (ENS) in Paris created multiple vortices by stirring a 87Rb BEC with a laser beam “stirring stick” that repelled the atoms by the AC Stark shift. The BEC was released from the trap and allowed to expand before imaging in order to allow the researchers to see the tiny vortex cores. With this laser-stirring technique, they were able to create lattices of up to 12 vortex cores within their condensates, pictured in Fig. 5 (a), showing the effects of increasing the amount of angular momentum given to the BEC.

More recently, Wolfgang Ketterle’s group at MIT has used this laser-stirring technique with much larger numbers of condensed atoms. This allowed the group to produce vortex lattices of up to ~130 cores, as shown in Fig. 5 (b). These striking images of vortex lattices provide a means of accurately studying vortex lattice crystallization, melting, and defects. In both the BEC and optical vortex cases, the stability and lifetime of vortices are dependent upon the sign and strength of the non-linearities present.

In a second experiment, the MIT group swept a laser beam through a BEC, creating vortices in the wake of the moving beam. By overlapping this BEC with a second vortex-free condensate, they were able to see an interference pattern with the expected signatures of a vortex, a localized dislocation of interference fringes in the region of the phase singularity. Analogue measurements in optics are a standard method by which vorticity in an optical beam may be determined.

Beyond vortices

Many experiments in BEC research involve concepts that are different from but related to vortices. For example, dark solitons, an important area of study in nonlinear optics, have also been created and observed in BECs, as shown in Fig. 6. Optical dark solitons may decay by non-linear instabilities. One can get similar dynamics in BECs with a single significant difference: Dark solitons in a BEC may decay into vortex rings rather than lines. Vortex rings do not exist in optics. To picture a vortex ring, first think about a vortex line. If you take the two ends of the vortex line, bend the line into a loop (still within the BEC) and connect the ends, you will have formed a vortex ring flow pattern. Cores of vortex rings were observed at JILA in BECs by allowing engineered dark solitons in the BEC to decay via nonlinear instabilities into more stable structures.

Conclusions

Nonlinear optics has long been a fascinating field covering a wide range of phenomena. BEC is now allowing us to see nonlinear effects with coherent atoms instead of photons. In many cases these nonlinear effects are quite similar to their optical counterparts, but they also often have their own unique signatures and surprises.

References

1. Bose-Einstein condensation in a dilute gas was first reported by M.H.Ander son et al., Science 299, 198-201 (1999). A summary of groups that have since obtained BEC can be found at http://amo.phy.gsu.edu/bec.html.

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