The goal of ellipsometry is to determine the optical and structural constants of thin films and flat surfaces based on the measurements of the ellipse of polarization in reflected or transmitted light. In the absence of birefringence and optical activity, a flat surface, a single-layer film, or a thin-film stack may be characterized by the complex reflection coefficients \( r_p = |r_p| \exp(i \phi_p) \) and \( r_s = |r_s| \exp(i \phi_s) \) for \( p \)- and \( s \)-polarized incident beams, as well as by the corresponding transmission coefficients \( t_p = |t_p| \exp(i \phi_t) \) and \( t_s = |t_s| \exp(i \phi_t) \).

Strictly speaking, an ellipsometer is a device that measures the complex ratios \( r_p/r_s \) and/or \( t_p/t_s \). The amplitude ratios are usually deduced from the angles \( \psi_p \) and \( \psi_s \), defined as \( \tan \psi_p = |r_p|/|r_s| \) and \( \tan \psi_s = |t_p|/|t_s| \). In practice, measuring the individual reflectivities \( R_p = |r_p|^2 \), \( R_s = |r_s|^2 \) or transmissivities \( T_p = |t_p|^2 \), \( T_s = |t_s|^2 \) does not require much additional effort. Measuring the individual phases, of course, is difficult, but the relative phase angles \( \phi_p - \phi_s \) and \( \phi_t - \phi_s \) can be readily obtained by ellipsometric methods. The values of \( R_p \), \( R_s \), \( \phi_p - \phi_s \), \( \psi_p \), \( \psi_s \), \( \phi_t - \phi_s \) and \( \phi_t \) may be measured as functions of the angle of incidence, \( \theta \), or as functions of the wavelength of light, \( \lambda \), or both.

The results of ellipsometric measurements are fed to a computer program that searches the space of unknown parameters to find agreement between the measured data points and theoretical calculations.

The unknown parameters of the sample usually include thickness, refractive index, and absorption coefficient of one or more layers. In general, the larger the collected data set is, the more accurate will be the estimates of the unknown parameters, or the greater will be the number of unknowns that can be estimated. The relationship between the measurable and the unknowns is usually nonlinear, and there is no a priori guarantee that the various measurements on a given sample are independent of each other, nor that a given set of measurements is sufficient for determining the unknowns. Powerful numerical algorithms exist that search the space of unknown parameters and find estimates that closely reproduce the measured data.

**The Nulling Ellipsometer.** Figure 1 is the diagram of a conventional nulling ellipsometer. The quasi-monochromatic light of wavelength \( \lambda \) enters a rotatable polarizer whose transmission axis may be oriented at an arbitrary angle \( \rho_p \) relative to the \( X \)-axis. The polarizer’s output is thus a collimated, linearly polarized beam of light with an adjustable E-field orientation. This beam goes through a quarter-wave plate (QWP) whose fast and slow axes are fixed at \( \pm 45^\circ \) to the \( X \)-axis. (The QWP imparts a 90° relative phase shift to the E-field components along its axes.) The beam emerging from the QWP has equal amplitudes along \( X \) and \( Y \), that is, \( |E_x| = |E_y| \). The phase difference between these E-field com-
components is adjustable in accordance with the following relation: \( \Phi_s - \Phi_r = 2(\rho_p - 45°) \).

Reflection from the sample imparts a phase difference \( (\Phi_{rp} - \Phi_p) \) to the \( p \)- and \( s \)-components of the beam, which may be cancelled out by properly selecting the polarizer angle \( \rho_p \). At this point the reflected beam is linearly polarized, with its parts a phase difference \( (\Phi_{rp} - \Phi_p) \) and \( |r_p| \), being proportional to \( |r_p| \) and \( |r_s| \). In the reflected path the analyzer, whose orientation is block the light that would otherwise reach the detector. Thus by measuring the values of \( \rho_s \) and \( \rho_p \) that null the detector's signal, one obtains the amplitude ratio \( |r_p|/|r_s| \) and the relative phase \( (\Phi_p - \Phi_{rs}) \) of the sample's reflection coefficients.

Measuring the sample reflectivities \( R_p, R_s \), using a nulling ellipsometer is straightforward: all one needs to do is monitor the detector signal \( S \) at \( \rho_s = 0° \) and \( 90° \). Calibration requires removing the sample and aligning the arms of the ellipsometer with each other (i.e., \( \theta = 90° \)), in which case the light from the source goes through the entire system and yields a detector signal corresponding to a 100% sample reflectivity. Optical power fluctuations could be countered by splitting off a small fraction of the beam at the source and monitoring its variations with an auxiliary detector. The signal from the auxiliary detector is subsequently used to normalize the reflectivity signals.

Needless to say, the same type of measurements as discussed above, when performed on the transmitted beam, yield the values of \( T_p, T_s, (\Phi_{tp} - \Phi_{ts}) \) and \( \psi_t \).

**Thin Film on Transparent Substrate.** Figure 2 shows a sample consisting of a thin absorbing layer on a glass substrate. To allow the transmitted beam to exit the substrate without a change in its state of polarization, and also to eliminate spurious reflections, an anti-reflection coated hemispherical substrate is assumed. The 25 nm-thick film has complex index of refraction \( n + ik = 4.5 + 1.75i \), and the substrate's refractive index is \( n_0 = 1.5 \). Computed values of the sample's reflection and transmission characteristics at \( \lambda = 633 \text{ nm, } \theta = 60° \) are: \( R_p = 29.63\%, R_s = 74.83\%, \Phi_{tp} - \Phi_{ts} = 3.95°, \psi_p = 32.18°, T_p = 24.13\%, T_s = 6.96\%, \Phi_{tp} - \Phi_{ts} = 1.50°, \psi_t = 61.76° \).

We examine the sensitivity of ellipsometric measurements to variations of the sample parameters. For example, if the refractive index \( n \) of the film is varied in the range from 4.0 to 5.0, the various characteristics of the sample vary as in Figure 3. (The variations are relative to the nominal characteristics evaluated at \( n = 4.5 \).) It is seen that \( R_p, R_s, \psi_p, \psi_t \) are more sensitive to changes of \( n \) than \( T_p, T_s \). \( (\Phi_{tp} - \Phi_{ts}) \) and \( (\Phi_{tp} - \Phi_{ts}) \). Similarly, Figure 4 shows variations of the sample characteristics with changes in \( k \). Here \( T_p, (\Phi_{tp} - \Phi_{ts}) \) and \( (\Phi_{tp} - \Phi_{ts}) \) are the more sensitive probes of \( k \) than \( R_p, R_s, T_p, T_s, \psi_p, \psi_t \). Figure 5 shows variations of the sample characteristics with a changing film thickness \( d \) in the range from 20 nm to 30 nm. In this case \( \psi_t \) and, to some extent, \( \psi_p \) are insensitive to \( d \), but the remaining characteristics are quite sensitive. 7

When all the components of the system are assumed to be perfect, the ellipsometer is sensitive enough to accurately de-

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**Figure 2.** A 25 nm-thick film of refractive index \( n + ik = 4.5 + 1.75i \) is deposited on a hemispherical glass substrate \( (n_0 = 1.5) \). The probe beam has \( \lambda = 633 \text{ nm and is incident at } \theta = 60° \). To avoid complications arising from reflections/losses at the substrate bottom, this hemispherical surface is antireflection-coated.

**Figure 3.** Variations of the reflection and transmission characteristics of the sample of Figure 2 at \( \lambda = 633 \text{ nm, } \theta = 60° \), when the film's refractive index \( n \) is varied from 4.0 to 5.0. The changes are relative to the nominal values obtained with \( n = 4.5 \).
termine the unknown sample parameters. In practice, however, no measurement system is perfect: the polarizer and the analyzer have a finite extinction ratio, allowing a small fraction of the undesirable E-field component to pass through; the quarter-wave plate's retardation deviates from 90°, and the beam that illuminates the sample is not an ideal plane wave, but has a finite diameter. Moreover, when the beam is focused on the sample to provide a reasonable spatial resolution, the focused cone of light contains a range of incidence angles, resulting in measured values that are averages over these angles. One consequence of such system imperfections is that, in the “null condition,” a minimum amount of light would still reach the detector. Another consequence is the limited accuracy with which the various reflection/transmission characteristics of the sample are measured.

**Performance of the Nulling Ellipsometer.** For the system depicted in Figure 1 we show in Figure 6 computed plots of the detector signal $S$ versus the angle $\rho_p$ of the analyzer for several values of the polarizer angle $\rho_p$. The assumed focusing and collimating lenses are identical, having NA = 0.025, corresponding to a 3° focused cone at the sample. In Figure 6(a) the assumed system is perfect, while in Figure 6(b) errors are incorporated into the various components, namely, the assumed polarizer and analyzer have a 1:1000 extinction ratio, the angle of incidence on the sample is $\theta = 61°$, and the QWP's retardation is 87° while its axes are 1° away from the ideal 45° orientation.

The null in Figure 6(a) is achieved with $\rho_p = 47°$, $\rho_a = 32.2°$, yielding $\phi_{rp} - \phi_{rs} = 4°$ and $\psi_r = 32.2°$, as expected. Also the detector signals at $\rho_a = 0°$ and 90° are 0.296 and 0.748, which correspond to the correct values of $R_p$ and $R_s$. In practice, even in this ideal case with perfect components, the exact location of the null may not be easy to determine. This produces a certain degree of inaccuracy, depending on the available signal-to-noise ratio at the detector. In the case of Figure 6(b), where the assumed components have substantial errors, the minimum signal occurs at $\rho_p = 54°$, $\rho_a = 30°$, yielding $\phi_{rp} - \phi_{rs} = 18°$, $\psi_r = 30°$. The reflectivities in this case (obtained at $\rho_a = 9°$ and $\rho_a = 0°$ and 90°) are $R_p = 0.308$, $R_s = 0.727$. Considering the sensitivity curves in Figures 3-5, such huge errors are clearly unacceptable.

A more realistic situation might correspond to small system errors, for instance, when the polarizer and the analyzer have extinction ratios of 1:1000, the angle of incidence on the sample is off by 0.25° ($\theta = 60.25°$), and the QWP's retardation is 90.5° while its axes are misaligned by only 0.25°. In this case the minimum signal occurs at $\rho_p = 49°$, $\rho_a = 31.8°$, yielding $\phi_{rp} - \phi_{rs} = 8°$, $\psi_r = 31.8°$. The reflectivities (obtained at $\rho_p = 4°$ and $\rho_p = 0°$ and 90°) are $R_p = 0.291$, $R_s = 0.757$. It is thus clear that the nulling ellipsometer requires a high degree of accuracy in its components in order to achieve a reasonable level of confidence in the estimated sample parameters.

**Ellipsometry with a Variable Retarder.** Figure 7 shows a different
kind of ellipsometer consisting of a fixed polarizer, a variable retarder (e.g., a liquid crystal cell or a photo-elastic modulator), and a fixed differential detection module. None of these components need to be rotated or otherwise adjusted during measurements. The variable retarder provides a range of polarization states at the sample. For instance, the incident beam is p-polarized when the retardation $\Delta \phi$ is 0°, circularly polarized when $\Delta \phi = \pm 90^\circ$, and s-polarized when $\Delta \phi = 180^\circ$. The detection module consists of a Wollaston prism with transmission axes fixed at $\pm 45^\circ$ to the plane of incidence, followed by a pair of identical photodetectors.

When the relative phase $\Delta \phi$ imparted by the retarder to the incident beam is continuously varied from 0° to 360°, the sum signal $S_1 + S_2$ oscillates between a maximum and a minimum value; these correspond to $R_p$ and $R_s$, although not necessarily in that order. At the same time, the normalized difference signal $(S_1 - S_2)/(S_1 + S_2)$ exhibits a peak-to-valley variation equal to $2 \sin (\Phi_{rp} - \Phi_{rs})$. The system of Figure 7 does not provide an independent measure of the other ellipsometric parameter, $\psi_r$. However, since $R_p$ and $R_s$ are directly measurable, $\psi_r$ is redundant.

In operating the system of Figure 7 it is not necessary to know the time-dependence of the retardation $\Delta \phi$, nor in fact does one need to know the specific value of $\Delta \phi$ at any point during the measurement. The maximum and minimum values of the sum signal and of the normalized difference signal contain all the necessary information. Unlike the nulling ellipsometer, this system does not require any adjustment of angles around a broad minimum; therefore, there is much less uncertainty about the measured values.

For the ideal system depicted in Figure 7, Figure 8(a) shows computed plots of the sum signal and the normalized difference signal versus the retardation $\Delta \phi$. The maximum and minimum values of the sum signal are 0.748 and 0.296, corresponding to $R_p$ and $R_s$. The normalized difference signal has a peak-to-valley variation of 0.1375, yielding $\Phi_{rp} - \Phi_{rs} = 3.94^\circ$.

In Figure 8(b) we have assumed some imperfection in the system components. Two cases are examined, one leading to the solid curves, the other to the dashed curves. In the former case the polarizer's extinction ratio is 1:100, the retarder axes are misaligned by 1°, the Wollaston prism has a 1:100 leak ratio between

**Figure 7.** Diagram of an ellipsometer based on a variable retarder and a differential detection module. The beam emerging from the polarizer is collimated and linearly polarized along the X-axis. The variable retarder's axes are fixed at $\pm 45^\circ$ to the XZ plane of incidence, while its phase is varied continuously from 0° to 360°. The light beam is focused on the sample through a low-NA lens, and the reflected beam is recollimated by an identical lens in the return path. The reflected beam is monitored by a differential detector consisting of a Wollaston prism (oriented at 45° to the plane of incidence) and two identical photodetectors. The sum of the detector signals $S_1 + S_2$ contains information about the sample reflectivities $R_p$ and $R_s$, while their normalized difference $(S_1 - S_2)/(S_1 + S_2)$ yields the relative phase $(\Phi_{rp} - \Phi_{rs})$. The detector signal $S$ versus the orientation angle $\rho_a$ of the analyzer in the nulling ellipsometer of Figure 1 with the sample of Figure 2. Different curves correspond to different values of the polarizer angle $\rho_p$. The total optical power of the unpolarized (or circularly polarized) beam emerging from the source is unity, the detector's conversion factor is 4, the incidence angle is $\theta = 60^\circ$, and the focusing and collimating lenses have NA = 0.025. In (a) the assumed system is perfect. In (b) there are departures from ideal behavior, namely, the polarizer and analyzer have a 1:100 extinction ratio, the angle of incidence is off by 1°, and the quarter-wave plate's retardation is 87° while its axes are 1° away from the ideal 45° orientation.
its two channels, and the angle of incidence $\theta$ is off by 0.5°. From the computed sum and difference signals we find $R_s = 0.290$, $R_p = 0.750$, and $\Phi_{sp} - \Phi_{rp} = 4.23°$. In the case of dashed curves in Figure 8(b) the assumed imperfections are large. Here the polarizer’s extinction ratio is 1:100, the retarder’s orientation angle is 43°, the angle of incidence is $\theta = 60.5°$, and the Wollaston prism leaks 2% of the wrong polarization into each channel. From the computed sum and difference signals the values of $R_p = 0.290$, $R_s = 0.749$, and $\Phi_{sp} - \Phi_{rp} = 4.6°$ are obtained. Obviously, the system of Figure 7 is quite tolerant of imperfections and misalignments; therefore, it is suitable for accurate determination of the sample parameters.

**References**

7. The computer simulations reported in this article were performed by MULTILAYER™ and DIFFRACT™; both programs are products of MM Research, Inc., Tucson, AZ.

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**Zoom Lens for a Digital Camera**

**BY J. BRIAN CALDWELL**

**Patent:** U.S. 6,014,268  
**Issued:** January 11, 2000  
**Title:** Two-Group Small Zoom  
**Example:** #9 of 11  
**Inventor:** Satoshi Yahagi  
**Assignee:** Fuji Photo Optical Co., Ltd.

A steady stream of new digital cameras offering improved resolution, smaller size and lower price is rapidly eroding the dominance of film cameras in the consumer photographic marketplace. This trend is reflected by an ever-increasing number of digital format lens designs seen in the patent literature. General characteristics of these designs typically include a very small image size, a telecentric or nearly telecentric exit pupil, and a ratio of overall system length to image diagonal that is much larger than designs intended for film. This month’s design, shown in Figure 1 and listed in Table 1, is a 2.8:1 zoom lens intended for a 1/4-inch (4.5mm diagonal) sensor. The construction is a two-group negative-positive type with the negative front group consisting of two negative elements followed by a positive meniscus element. This design is similar to

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**Figure 1.** 3.37mm - 9.44mm, f/2.4 - f/3.3 zoom lens for an electronic camera using a 1/4-inch sensor.